1. (2 pt.) Consider the language $L = \{a^n b^n : n > 0\}$. Is the string $122$ in $L$?

2. (3 pt.) Let $L_1 = \{a^n b^n : n > 0\}$. Let $L_2 = \{c^n : n > 0\}$. For each of the following strings, state whether or not it is an element of $L_1 L_2$:
   a) $\epsilon$
   b) $abbcc$
   c) $aabbcccc$

3. (3 pt.) Let $L = \{w \in \{a, b\}^* : |w| \equiv_3 0\}$. List the first six elements in a lexicographic enumeration of $L$.

4. (4 pt.) Are the following sets closed under the following operations?
   a) The language $\{a, b\}$ under concatenation.

   b) $L = \{w \in \{a, b\}^*\}$ under reverse.
5. (6 pt.) For each of the following statements, state whether it is True or False. Prove your answer.
   a) \( \forall L_1, L_2 \ (L_1 = L_2 \iff L_1^* = L_2^*) \).

   b) Every infinite language is the complement of a finite language.

6. (6 pt.) Let \( \Sigma = \{a, b, c\} \). Let \( S \) be the set of all languages over \( \Sigma \). Let \( f \) be a binary function defined as follows:

   \[
   f : S \times S \rightarrow S \\
   f(x, y) = x - y
   \]

   Answer each of the following questions and defend your answers:
   a) Is \( f \) one-to-one?

   b) Is \( f \) onto?

   c) Is \( f \) commutative?
7. (6 pt.) Build a deterministic FSM for each of the following languages:
   
a) \( \{ w \in \{a, b\}^*: \text{every } a \text{ in } w \text{ is immediately preceded and followed by } b \} \).

b) \( \{ w \in \{a, b\}^*: \text{w does not end in } ba \} \).
8. (6 pt.) Consider the following NDFSM $M$:

For each of the following strings $w$, determine whether $w \in L(M)$:

a) $aabbba$

b) $bab$

c) $baba$

9. (4 pt.) Write a regular expression to describe each of the following languages:

a) $\{w \in \{0, 1\}^* : w$ does not have $001$ as a substring$\}$. 

b) $\{w \in \{a, b\}^* : \#_a(w) \equiv_3 0\}$. 
10. (4 pt.) Let $L$ be the language accepted by the following finite state machine:

Indicate, for each of the following regular expressions, whether it correctly describes $L$:

a) $(a \cup ba)bb^*a$.

b) $(\varepsilon \cup b)a(bb^*a)^*$.

c) $ba \cup ab^*a$.

d) $(a \cup ba)(bb^*a)^*$.

11. (6 pt.) Let $L = \{w \in \{a, b\}^* : \text{every } a \text{ in } w \text{ is immediately followed by at least one } b\}$.

a) Write a regular expression that describes $L$.

b) Write a regular grammar that generates $L$.

c) Construct an FSM that accepts $L$. 